MPRI PRFA

Proof assistants

Yannick Forster

Théo Winterhalter

Goals

Ensure you are familiar enough with one proof assistant (Coq) so that you can

- 🧑‍🎓 use Coq in other courses
- 👩‍💻 use Coq in an internship
- 👀 learn other proof assistants or become an expert Coq user via self study
- 🧙‍♂️ ultimately use or study proof assistants as part of a PhD

We also cover meta-theory, in particular dependent type theory

(more in 2-7-1)

Important information on the course webpage

https://mpri-prfa.github.io/

Organisation

8 lectures

30 min | 1h30 | Break (10 min)

Review | Lecture + live coding | Practice

Including advanced *optional* exercises (50 min)

Self-practice during the week

Teachers

Yannick Forster

Théo Winterhalter

Project

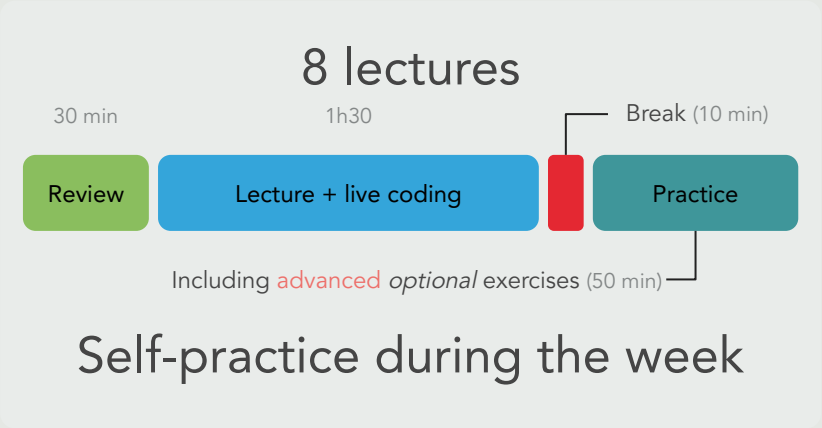
One big Coq exercise with files to complete and a report to write

+

3h exam

you may only bring *one handwritten* A4 page (both sides)

Proving is Programming



Project

One big Coq exercise with files to complete and a report to write

Learning how to program takes a lot of practice

Every part of this course tries to help you practice

- Practical lectures with live coding
- Practice after the lecture
- Practice at home
- Practice during the project
- Self-evaluation

Keeping in touch

Join the course's Discord server

https://discord.gg/Cxdxw3Tr

Discuss exercises, the project, other proof assistants…

We'll post about internships too!

Frequently asked questions will be added to… the FAQ

Useful resources

Coq official website
coq.inria.fr

Links to everything you may need related to Coq

Coq documentation
coq.inria.fr/doc/V8.18.0/refman/

Tactic index, command index and more…

Coq discourse
coq.discourse.group

Forum for announcements and questions available in several languages

Coq Zulip
coq.zulipchat.com

Chat where most of Coq discussions happen nowadays

Outline of the course

Subject to change
The course webpage is authoritative

23 Sept.	Intro.
30 Sept.	Inductive types.
7 Oct.	Proof terms and meta-theory.
14 Oct.	Mathematical modelling. Automation.
21 Oct.	Equality.
28 Oct.	Advanced elimination / induction.
4 Nov.	Tactics and meta-programming.
18 Nov.	Dependent functional programming.

Important dates

early Oct. Project handout

14 Nov. Project deadline

25 Nov. Exam

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Have efore?

https://framaforms.org/registration-for-mpri-course-2-7-2-prfa-1726732614

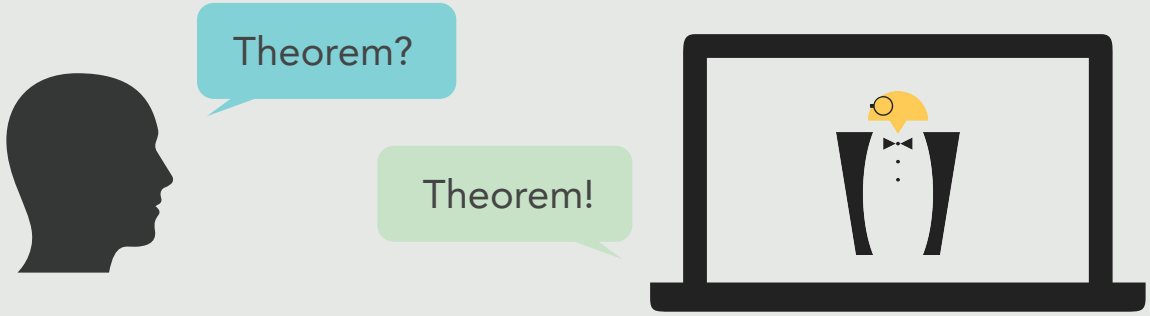
Today

Short introduction on proof assistants

Getting acquainted with Coq

Proving things in propositional logic

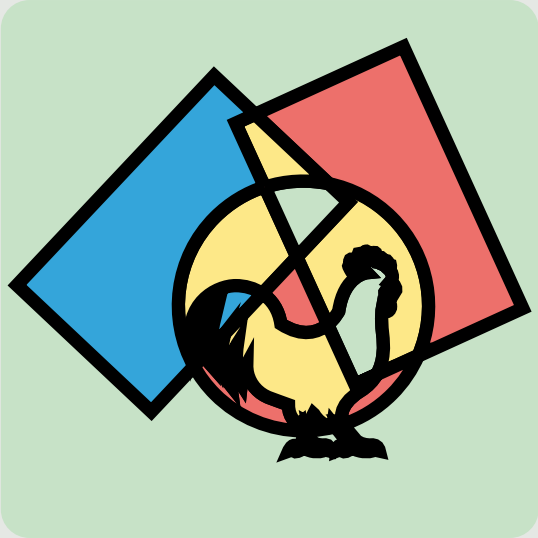
What is a proof assistant?



A piece of software for stating and proving mathematical theorems

It helps you build proofs interactively by giving you feedback,
inferring missing information, and, crucially, checking proofs!

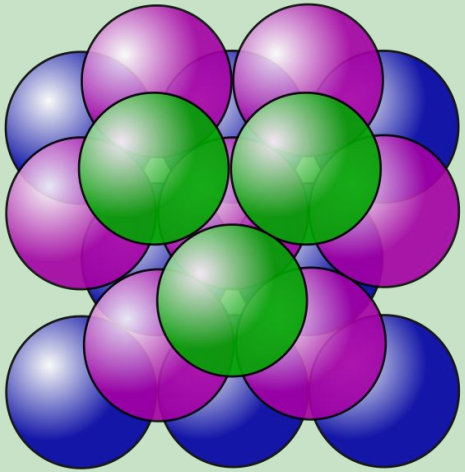
Why a proof assistant?



Mathematics

Program verification

Kepler conjecture



Source: wikipedia

1611	Conjecture by Johannes Kepler
1998	Proof by Tom Hales 300 pages, 400k lines of code
1999	12 reviewers, 99% certain
1999 2002	Continued reviewing, still 99% certain
2005	Published in Annals of Mathematics, “without complete certification from the referees”
2003- 2015	Formal proof project in Isabelle and HOL light “Flyspeck”
2017	Published formal proof in Forum of Mathematics

Four colour theorem

1852 Conjecture by Francis Guthrie

1879 Initial proof by Alfred Kempe

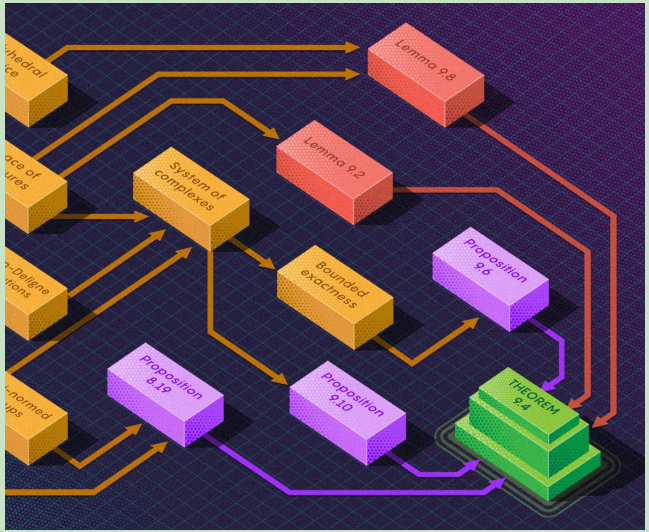
1890 Percy Heawood finds a mistake
(5 colour theorem)

1976 Proof by Kenneth Appel and Wolfgang Haken
Proof idea: find an “unavoidable”, “reducible” set of configurations
Reducibility: Checked by computer, took 2 days
Unavoidability: 400 pages of microfiche, checked manually by Appel, Haken, and Haken’s teenage daughter Dorothea Blostein

1981 Mistakes found by Master’s student but fixed

2004 Proof in Coq by Georges Gonthier with Benjamin Werner

The liquid tensor experiment



Samuel Velasco/Quanta Magazine; Johan Commelin

https://www.quantamagazine.org/lean-computer-program-confirms-peter-scholze-proof-20210728/

2019 July: Scholze works out proof of central theorem of Scholze-Clausen liquid mathematics, mainly in his head

2019 Proof is written up, but Scholze is unsure about parts

2020 Scholze writes post on Kevin Buzzard's blog about "liquid tensor experiment", lead by Johan Commelin: A mechanisation of the proof in Lean

Working mode: Commelin works on the main proof, technical lemmas are outsourced to community via online chat

2021 May: Main argument mechanised

2022 July 14: complete proof mechanised

BusyBeaver(5)

https://www.quantamagazine.org/amateur-mathematicians-find-fifth-busy-beaver-turing-machine-20240702

https://www.lemonde.fr/sciences/article/2024/07/17/mathematiques-le-defi-du-castor-affaire-resolu_6251337_1650684.html

1962 Tibor Radó introduces “The busy beaver game”: BB(n) is the maximal number of steps a Turing machine with n states can take

1966 Allen Brady discovers 4 state machine taking 107 steps

1974 Brady proves BB(4) = 107

But there are 17 trillion possible 5 state Turing machines…

1989 Heiner Marxen and Jürgen Buntrock find 5 state machine taking 47,176,870 steps

2020 Scott Aronson conjectures BB(5) = 47,176,870

2021 Busy beaver challenge started by Tristan Stérin

2024 Coq proof of BB(5) = 47,176,870
by anonymous contributors

Compiling 1 million random C programs will result in miscompilation unless
Optimisations are disabled
Or the compiler is formally verified
John Regehr

CompCert: Fully verified C compiler with optimisations

More verified software: CakeML, sel4, FiatCrypto, Google boring SSL, …

Proof assistants can help when…

- … proofs are too big to be reviewed
- … proofs rely on complicated computer programs
- … proofs are too complicated to be trusted by their authors
- … proofs are contributed by anonymous hobbyist mathematicians
- … computer programs are too complicated to be trusted
- … students want to develop a deeper understanding what is a proof

We can help…

- … you to get started learning in the lectures
- … you to practice at home
- … you to assess your current level constantly
- … deepen your understanding of Coq – advanced exercises for all levels

Proving is programming: This course will take you more time than others!

Why Coq?

All proof assistants are beautiful!

But we have 8 x 3 hours to teach you

Option 1:

Reach limited proficiency in several proof assistant without understanding concepts deeply

Agda

Idris

LEAN

Goals

🧑‍🎓 use PAs in other courses

👨‍💻 use a PA in an internship

👀 learn other proof assistants or become an expert PA user via self study

🧙‍♂️ ultimately use or study proof assistants as part of a PhD

Why Coq?

All proof assistants are beautiful!

But we have 8 x 3 hours to teach you

Option 1:

Reach limited proficiency in several proof assistant without understanding concepts deeply

Option 2:

Reach very good proficiency in one proof assistant, be able to learn others on your own

Agda

Idris

LEAN

Let's get started!

Live-coding with Coq

Resulting file will be available on the course webpage

Please close your laptops and try to follow the lecture.
If you absolutely need your laptop, please sit in the last row.

Practice with Coq

Expression	Meaning	Prove it	Use it
`P -> Q`	implication	`intro`	`apply`
`P /\ Q`	conjunction	`split`	`destruct`
`P \/ Q`	disjunction	`left, right`	`destruct`
`~ P`	negation	`intro`	`apply`
`x = y`	equality	`reflexivity, f_equal`	`rewrite`
`0 + 2`	addition	`simpl`	
`n`	natural numbers	`constructor`	`induction`

Installation instructions are on the course webpage (in case you didn't instal Coq already)
https://mpri-prfa.github.io/#installing-coq