

Self-evaluation

7 November 2025

You are invited to hand in your solutions to help us understand the current state. You are encouraged to write your name on the top right, but it is not mandatory – it would be very helpful for us, and we will give individual feedback if we have your name.

Hint: Solve exercises that are easy for you first. Exercises are not necessarily in order of hardness.

Relevant constructors are

$\text{or_introl} : \forall \{A\} B, A \rightarrow A \vee B$	$\text{or_intror} : \forall A \{B\}, B \rightarrow A \vee B$
$\text{conj} : \forall \{A B\}, A \rightarrow B \rightarrow A \wedge B$	$I : \text{True}$
$\text{inl} : \forall \{A\} B, A \rightarrow A + B$	$\text{inr} : \forall A \{B\}, B \rightarrow A \vee B$

Question 1

Prove the following lemma using a proof script. Do not use `discriminate`, `congruence`, `inversion`.

Lemma `0_neq_1` : `0 <> 1`.

Proof.

Question 2

Given $A : \mathbf{Prop}$ and $B : \mathbf{Prop}$, consider the two types $T1 := A \vee B$ and $T2 := A + B$.

Define a function of type $T1 \rightarrow T2$ or explain why it is not possible.

Define a function of type $T2 \rightarrow T1$ or explain why it is not possible.

Question 3

Give the type of the induction principle for the following predicate:

```
Inductive Forall {A : Type} (P : A → Prop) : list A → Prop :=  
| Forall_nil : Forall P nil  
| Forall_cons : ∀ (x : A) (l : list A), P x → Forall P l → Forall P (x :: l).
```

Question 4

Write down the goal (assumptions and conclusion) after executing the following proof script:

```
Inductive bintree A :=  
| bt_leaf (a : A)  
| bt_node (a : A) (l r : bintree A).  
  
Fixpoint bt_map {A B} (f : A → B) (t : bintree A) : bintree B :=  
  match t with  
  | bt_leaf _ a ⇒ bt_leaf _ (f a)  
  | bt_node _ a l r ⇒ bt_node _ (f a) (bt_map f l) (bt_map f r)  
  end.  
  
Lemma bt_map_id A (t : bintree A) :  
  bt_map (fun x ⇒ x) t = t.  
Proof.  
  induction t as [a | a l ihl r ihr].  
  — reflexivity.  
  — cbn. rewrite ihl.
```

Question 5

Write down a proof term for $\forall A B : \text{Prop}, A \vee B \rightarrow ((A \wedge \text{True}) \vee B) \vee \text{False}$.